$$Y = 1/(\omega_1{}^2 + \omega_2{}^2)(-2(1 + 2K)[(1/\omega_1)\sinh\omega_1 t - (1/\omega_2)\sin\omega_2 t]X_0 + [(\omega_1{}^2 - 2K + 3)\cosh\omega_1 t + (\omega_2{}^2 + 2K - 3)\cos\omega_2 t]Y_0 - 2(\cosh\omega_1 t - \cos\omega_2 t)\dot{X}_0 + \{[(\omega_1 - 2K - 1)/\omega_1]\sinh\omega_1 t + [(\omega_2{}^2 + 2K + 1)/\omega_1] \times \sin\omega_2 t\}\dot{Y}_0) \quad (12)$$

$$Z = (\dot{Z}_0/\omega_3)\sin\omega_3 t + Z_0\cos\omega_3 t \quad (13)$$

where $X_0Y_0Z_0\dot{X}_0\dot{Y}_0\dot{Z}_0$ = the initial position and velocity components at TPI.

$\dot{X}_0\dot{Y}_0$ and $\dot{Z}_0$ will not generally result in a rendezvous. It is therefore necessary to determine the desired velocities $\dot{X}_{od}\dot{Y}_{od}\dot{Z}_{od}$ that will insure rendezvous. Since the terminal guidance equations are functions of time, it is practical to specify a nominal time T when rendezvous will occur [i.e., $X(T) = Y(T) = Z(T) = 0$]. Since there are three equations and three unknowns the desired velocities can easily be computed assuming that the initial position is known (Table 1).

To find the (TPI) $\Delta\mathbf{V}_{1_i}$ needed, the initial velocities are subtracted from the desired velocities (Fig. 1):

$$\Delta\mathbf{V}_{1_i} = (\dot{X}_{od} - \dot{X}_o)e_X + (\dot{Y}_{od} - \dot{Y}_o)e_Y + (\dot{Z}_{od} - \dot{Z}_o)e_Z \quad (14)$$

The braking maneuver needed to negate the satellite's relative velocity at rendezvous can be written as follows:

$$\Delta\mathbf{V}_{2_o} = -\dot{X}(T)e_X - \dot{Y}(T)e_Y - \dot{Z}(T)e_Z \quad (15)$$

In a typical trajectory, rendezvous would occur at some point in the orbit rather than at the L_2 point. Using points 1 and 2 for rendezvous (Fig. 1) would result in the smallest ΔV requirement.

Rendezvous at POI would not only entail a braking maneuver as shown in Eq. (12), but an injection maneuver as well, and can therefore be written

$$\Delta\mathbf{V}_{2_A} = \Delta\mathbf{V}_{2_o} + \Delta\mathbf{V}_{2_i} \quad (16)$$

To show how representative the ΔV costs are, one can compare them with the values obtained by General Electric, since their analysis is the only quantitative trajectory study for establishing a Halo orbit presently available.

As depicted in Table 2, the total cost for the moon $-L_2$ transfer computed in this study is 210 fps greater than General Electric's results. The difference in total cost for the moon $-L_2$ transfer can be explained in several ways. The most important reason is that the method described in this paper is a preliminary study and by no means optimizes the ΔV cost. Also the orbits and the method of injection are completely different. Nevertheless by performing the analysis in the manner previously described reasonable and representative values are obtained.

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An Expression for Ring-Baffle-Slosh-Damping under Reduced Gravity Conditions

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Nomenclature

a = tank radius
C_D = baffle drag coefficient
d = depth of baffle below free surface
N_R = Reynolds number
P_d = period parameter
U_m = maximum fluid velocity past baffle
W = baffle width
α = $(2aW - W^2)/a^2$
γ = damping factor
η_w = maximum wave amplitude at tank wall
μ, ρ = propellant viscosity and density, respectively
τ = period of slosh oscillations
Φ = potential function

When an orbiting vehicle containing liquid propellant is stabilized by an active control system, liquid sloshing under low gravity conditions occurs. A great deal of work, both experimental and analytical, has been devoted to determining the frequencies, forces, and moments of the sloshing propellant. Slosh damping is an important physical parameter about which only a little information is now available. In most cases it is important to be able to predict the damping coefficient of a particular liquid-tank system and where this capability is insufficient, slosh baffles must be sized to provide the correct amount. For predicting damping in smooth-walled tanks (tanks without baffles), the experiment data obtained for high-gravity situations are applicable as long as the interface of the liquid is nearly flat. Where the free surface is moderately curved, some information on slosh damping is available.[1] There is a need, however, for a method for calculating damping coefficients for baffled containers.

The most widely used expression for predicting damping by flat ring baffles was derived by J. W. Miles,[2] who combined drag coefficient data obtained by Keulegan and Carpenter[3] with the potential flow solution for sloshing in a cylindrical

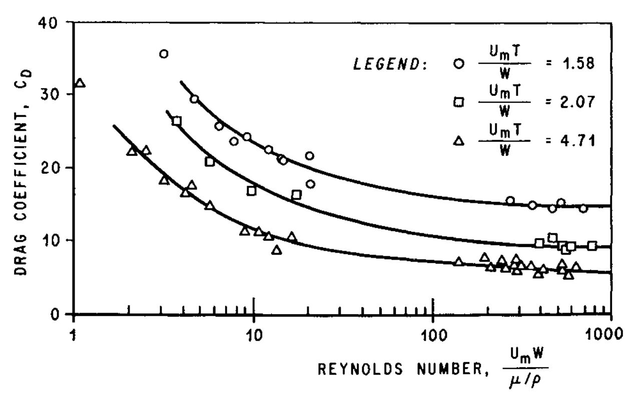

Fig. 1 Drag coefficient as a function of Reynolds number for various period parameters.

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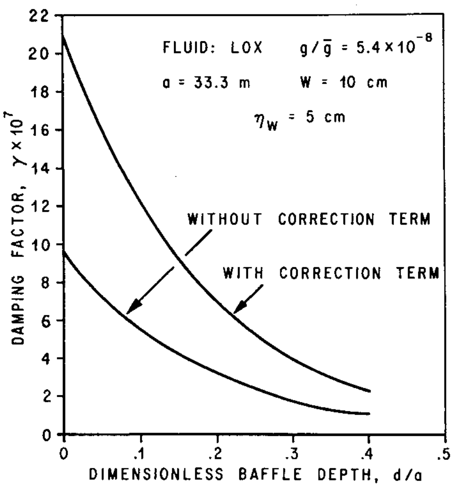

Fig. 2 Effect of Reynolds number correction term.

tank. The result was an expression for the damping provided by one flat ring baffle under high-gravity conditions. One of the most striking differences between high- and low-gravity sloshing is the period of oscillation. In a large tank, like the S-IVB stage of the Saturn V, periods of several hundred seconds[4] have been observed when the vehicle was under vent thrust in Earth orbit. For situations like this, the liquid interface is nearly flat and surface tension effects are small because of the size of the tank. The body forces associated with the applied thrust predominate, and yet are so small that the motion is very slow. A careful examination of these factors reveals that the technique used by Miles is still applicable for predicting the damping with reasonable accuracy provided that the drag coefficient data similar to that of Ref. 3 can be found for much lower Reynolds numbers.

Analysis

The experimental investigation reported in Ref. 5 was conducted to obtain drag coefficient data for Reynolds numbers from 1 to 1000. Analysis of the results yielded a functional relationship among the pertinent parameters: the drag coefficient, Reynolds number and the period parameter as shown in Fig. 1.

This functional relationship may be expressed mathematically in explicit form as follows:

$$C_D = 15P_d^{-1/2}\exp(1.88/N_R^{0.547}) \tag{1}$$

where C_D denotes the drag coefficient, $P_d = U_m\tau/W$ the period parameter, and $N_R = U_mW\rho/\mu$, the Reynolds number. Equation (1) was derived from a curve fitted by eye to the experimental data for Reynolds number ranging from 2 to 1.4×10^4.

According to Miles' analysis,[2] an expression for damping, γ, in terms of drag coefficient C_D, velocity potential ϕ, baffle geometrical parameter α, and wave amplitude parameter, η_w/a may be written

$$\gamma = C_D\phi(\eta_w/a)\alpha \tag{2}$$

where $\alpha = (2aW - W^2)/a^2$. In the expression for damping, γ is equivalent to the logarithmic decrement divided by 2π. For the particular case of a right circular cylinder filled with liquid and having a single ring baffle well beneath the liquid free surface, Eq. (2) was shown to become

$$\gamma = 0.5\exp(-5.52d/a)\alpha C_D(\eta_w/a) \tag{3}$$

where d/a denotes the baffle depth parameter.

Assuming that Miles' argument for adapting the flat plate results for ring damping is also valid for low Reynolds numbers, Eq. (1) may be substituted into Eq. (3):

$$\gamma = 7.5\alpha(\eta_w/a)P_d^{-1/2}\exp(N_R^{1.88/0.547} - 5.52d/a) \tag{4}$$

By including an additional term for Reynolds number in the exponent of Eq. (4) the range of application of the damping coefficient γ has been extended to low Reynolds numbers. In Fig. 2, damping factor is presented as a function of fluid depth over the baffle for both a case including the correction term and one without it. As can be seen by comparison of the two curves for large tanks at low-g levels, the relative effect of Reynolds number can be significant. The equation should not be applied to cases in which the interface is highly curved since the potential solution used in the analysis is not applicable, a good indication of the degree of surface curvature is provided by Bond number; high Bond number—interface flat, low Bond number—highly curved. However, for Bond numbers greater than 50 and Reynolds numbers smaller than 100, this expression should provide accurate results.

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Thermionic Reactor Ion Propulsion Spacecraft for Unmanned Outer Planet Exploration*

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THE nuclear thermionic reactor power system is one of the leading nuclear power system candidates for electric propulsion applications for unmanned missions to Jupiter, Saturn, Uranus, and Neptune. Nuclear thermionic power systems (like solar power systems) consist of many static power conversion modules arranged to tolerate module failures. The

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